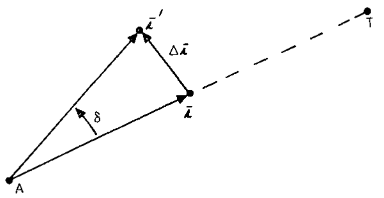

Fig. 1 Refraction error.

Now in accordance with Eq. (1), the erroneous LOSR is

$$\bar{\Omega}' = \bar{\imath}' \times \dot{\bar{\imath}}' \qquad (4)$$

Substituting Eq. (2) into (4)

$$\begin{aligned}\bar{\Omega}' &= (\bar{\imath} + \Delta\bar{\imath}) \times (\dot{\bar{\imath}} + \Delta\dot{\bar{\imath}}) \\ &= \bar{\Omega} + \Delta\bar{\imath} \times \dot{\bar{\imath}} + \bar{\imath} \times \Delta\dot{\bar{\imath}}\end{aligned} \qquad (5)$$

neglecting $\Delta\bar{\imath} \times \Delta\dot{\bar{\imath}}$ as second order. The error in the LOSR is

$$\Delta\bar{\Omega} = \bar{\Omega}' - \bar{\Omega} = \Delta\bar{\imath} \times \dot{\bar{\imath}} + \bar{\imath}\Delta \times \dot{\bar{\imath}} \qquad (6)$$

From Eq. (1), $\dot{\bar{\imath}} = \bar{\Omega} \times \bar{\imath}$, so the first term in Eq. (6) becomes

$$\Delta\bar{\imath} \times (\bar{\Omega} \times \bar{\imath}) = -\bar{\imath}(\Delta\bar{\imath}\cdot\bar{\Omega}), \text{ since } \Delta\bar{\imath}\cdot\bar{\imath} = 0 \qquad (7)$$

The term in Eq. (7) lies along $\bar{\imath}$ and hence cannot be tracked as an error in the LOSR. Now a vector $\bar{\delta}$ can be constructed perpendicular to $\bar{\imath}$ and such that

$$\Delta\bar{\imath} = \bar{\delta} \times \bar{\imath} \qquad (8)$$

The second term in Eq. (6) then becomes

$$\begin{aligned}\bar{\imath} \times \Delta\dot{\bar{\imath}} &= \bar{\imath} \times (\dot{\bar{\delta}} \times \bar{\imath} + \bar{\delta} x \dot{\bar{\imath}}) \\ &= \dot{\bar{\delta}} - \bar{\imath}(\dot{\bar{\delta}}\cdot\bar{\imath}), \text{ since } \bar{\imath}\cdot\dot{\bar{\imath}} = \bar{\imath}\cdot\bar{\delta} = 0\end{aligned} \qquad (9)$$

The right-hand side of Eq. (9) is the derivative $\dot{\bar{\delta}}$ less the component along $\bar{\imath}$. Hence, combining Eqs. (6) and (9)

$$\Delta\bar{\Omega} = (\dot{\bar{\delta}})_{\perp} \text{ to } \bar{\imath} \qquad (10)$$

Let y and z be the radar axes perpendicular to the LOS, then in general

$$\bar{\delta} = \bar{\jmath}\delta_y + \bar{k}\delta_z \qquad (11)$$

and

$$\dot{\bar{\delta}} = \dot{\bar{\jmath}}\delta_y + \bar{\jmath}\dot{\delta}_y + \dot{\bar{k}}\delta_z + \bar{k}\dot{\delta}_z \qquad (12)$$

where

$$\dot{\bar{\jmath}} = \bar{\omega} \times \bar{\jmath}, \ \dot{\bar{k}} = \bar{\omega} \times \bar{k} \qquad (13)$$

and $\bar{\omega}$ is the rigid-body angular velocity of the tracking radar. Substituting Eq. (13) into (12) and suppressing the $\bar{\imath}$ component as required by Eq. (10)

$$\Delta\bar{\Omega} = \bar{\jmath}(\dot{\delta}_y - \omega_x\delta_z) + \bar{k}(\dot{\delta}_z + \omega_x\delta_y) \qquad (14)$$

where ω_x is the antenna roll rate around the LOS. Equation (14) gives the desired expression for the LOSR error generated by radome refraction errors. To carry the analysis further requires determination of $\dot{\delta}_y$, $\dot{\delta}_z$.

Now δ_y, δ_z are functions of the location on the radome where the beam pierces the radome. Typically, they are expressed as functions of two angular coordinates which determine the LOS attitude, e.g., azimuth and elevation E, A. Thus,

$$\delta_y = \delta_y(E,A), \qquad \delta_z = \delta_z(E,A) \qquad (15)$$

Then in Eq. (14)

$$\begin{aligned}\dot{\delta}_y &= (\partial\delta_y/\partial E)\dot{E} + (\partial\delta_y/\partial A)\dot{A} \\ \dot{\delta}_z &= (\partial\delta_z/\partial E)\dot{E} + (\partial\delta_z/\partial A)\dot{A}\end{aligned} \qquad (16)$$

The partial derivatives are called the radome error slopes. These derivatives must be measured for a particular radome, along with the refraction errors Eq. (15). Also needed in Eqs. (14) and (16) are the rates ω_x, $\dot{E}$, $\dot{A}$. These can be determined for a particular antenna gimballing arrangement as a function of the aircraft yaw, pitch and roll rates and the antenna rates about the y and z axes.

Conclusions

An expression for the vector LOSR error generated by radome refraction errors and their slopes is given by Eq. (14). What is needed are measured or assumed refraction errors and their slopes as a function of radome location, and various rates such as $\dot{E}$, $\dot{A}$, and ω_x. Numerical solution generally requires a computer. These expressions have formed the basis for studying the effects of radome errors in airborne tracking systems.

A Digital Computer Study of System Modeling by Pulse Testing

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THE objective of this Note is to report on a method to reduce the data obtained from a pulse test by digital computer methods and to approximate the data functionals by Fourier series in order to facilitate Fourier transform calculations.

The gathering of information about the dynamic properties of physical systems constitutes a problem for eng neers in all fields. For systems or components with low natural frequencies or short test times, such as aircraft, sinusoidal testing to obtain dynamic data is too lengthy and very costly. For these systems the best test input is usually a transient or pulse input. These inputs are defined as identically zero for all values of time less than zero and time greater than some finite value. Therefore, the system output wil also die out in finite time. Only one test needs to be run and it usually can be done quickly. Many investigators[1–4] have reported on the difficulties that arise from the processing of physical data taken from pulse tests of aircraft performance. One basic difficulty was found by Lees[5] to be nonconvergence of the numerically (digitally) calculated Fourier transforms (transfer functions) from the sampled data of the input-output time histories. Various interpolating polynomials were used to obtain integrals of the time histories of the input and output. Stepped function, trapezoidal, and parabolic approximations were attempted with the same nonconverging result. He noted a 5% accuracy of the transfer function calculation at a frequency of only 5% of the sampling theorem limit. He also notes the classical "breakdown" phenomenon that usually is present at high

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frequencies. This author[6] used Fourier series approximations of the input and output time histories before performing the numerical Fourier transform calculations. It was expected by such a technique one could reduce the number of points picked off the time-response graphs without sacrificing overall accuracy. This technique then uses the continuous finite Fourier series approximations in the integration routine. Thus a smaller calculation increment may be used than the time increment n the original sampled data points.

The integration routine used was the Filon-trapezoidal method. This method was necessary because of the rapidly oscillating integrands at high frequencies. This technique was found to improve the transform calcu ations at high frequencies over the existing methods used but did not completely eliminate the "breakdown" phenomenon found by other investigators. The comparison between the pulse method and a strictly analog analyzer method (sinusoidal test ng) for experimental transfer functions showed no distinct accuracy advantage to either method

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Thermal Effects on Aircraft Elastic Mode Shapes

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THE day will come, probably before the end of this century, when aircraft will be flying at high supersonic and hypersonic speeds within the atmosphere. Such aircraft will undoubtedly be highly flexible, resulting in total-airframe orthogonal vibration modes with undamped natural frequencies of the same order of magnitude as the rigid-body short-period frequency. Such circumstances will likely give rise to a severe mode interaction, wherein the aerodynamic coupling between the low-frequency elastic modes and the rigid-body motion can result in dynamic instabilities which would otherwise not occur.[1] This interaction phenomenon is extremely sensitive to variations of, or uncertainties in knowledge of, the exact shapes of the elastic modes, and very high accuracy (5% or better) will be required in analytically determining the mode shapes to be used in dynamic stability analyses and stability augmentation control system synthesis.

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Such aircraft will be subjected to severe transient and steady-state aerodynamic heating. The portion of the generated heat that is convected to the aircraft surfaces can cause large nonuniform increases in structural temperatures, which in turn produce thermally-induced stresses in the structure and reduction in the elastic moduli. Both thermal stresses and lower elastic moduli contribute significantly to reduced structural stiffness and aggravated static and dynamic aerothermoelastic problems.

A fair amount of research has been done on the effects of nonuniform temperature distributions on effective stiffness and natural frequencies of structures (although, little in the last 5 or 6 years); a small sampling is referenced.[2–7] However, an extensive literature survey has turned up precious little work on the effects of nonuniform temperature distributions on the normal vibration mode shapes of even simple structural elements such as plates and beams, let alone complex aerospace vehicle structures or structural components. Only one reference was found which contains data on such effects; it presents only experimental data with no analytical method offered.[8] A rectangular, stainless steel, cantilevered lifting surface of rib and spar construction was subjected to transient chordwise heating with quartz lamps, and the first and second vibration mode shapes were determined from vibration data. Even though the maximum temperature (about 600°F) and maximum temperature difference between two chordwise points (about 200°F) were very modest when compared to what a hypersonic aircraft will encounter, the mode shapes varied as much as 20% from their unheated shapes.

As pointed out previously, 5% accuracy will be needed in mode shape determination; thus, the need is apparent for research on analytical methods for calculating elastic mode shapes in the presence of nonuniform temperature distributions. It is urged that sponsoring agencies give attention and support to filling this serious gap in knowledge and analytical methods in order to have the analysis and design tools at hand when the application arises, as it most certainly will in the not too distant future.

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